Influence of Free Stream Velocity on the Entrainment by Single- and Two-Phase Axisymmetric Jets

VISWANATH SUBRAMANIAN

University of Newcastle
N.S.W. 2308, Australia

RAMAN GANESH

Indian Institute of Technology
Bombay 400 076, India

INTRODUCTION

The study of two-phase jets is of great importance in designing pulverized fuel furnaces and in pollution control. An idea of the particle dispersion in a jet surrounded by a moving stream is extremely useful in minimizing the spread of automobile exhaust and smoke issuing from stacks. Two-phase jet flows in a coflowing stream also find use in aerosols, rocket propulsion, spraying and dusting of agricultural crops and flue gases. Looking at the fields in which one comes across particle-laden jets it is surprising how so little work has been done in this area.

The study of single-phase jets, axisymmetric and plane has long since been done but these results do not have much value in predicting, for example, the characteristics of an air-coal jet because coal particles can make significant contributions to the mass, momentum and energy transfer of the two-phase jet.

The only detailed experimental work reported so far is that of Laats and Frishman (1970), who measured the velocity and concentration profiles of a corrundum/air jet. But their results are in the region not very close to the nozzle which is the area where ignition occurs in a combustion chamber. Moreover, calculations based on their velocity profile suggest a decrease in the mixing with increase in particle concentration, whereas Field (1963) has measured entrainment by a lycopodium/air jet using Ricou and Spalding's (1961) technique and found that it can increase, decrease or remain unaffected depending on the particle size.

Mixing rates of particles in gas for confined coaxial jets with conditions close to those of pulverized coal combustion and gasification processes have been reported by Memmot and Smoot (1978) who provided measurements of the gas velocity, particle mass flux and gas composition at various radial and axial locations downstream of the jet exit plane. Shinichi Yuu et al. (1978) studied the turbulent diffusion mechanism of particles in a round air jet paying particular attention to the relative velocity between particles and the fluid. Their results indicate that the particle diffusivity decreases with increase of the particle inertia. Velocity measurements on two-phase jets have also been reported by Levy and Lockwood (1981) and Wall et al. (1982).

Our previous work (1982a) on a two-phase round jet considered the entrainment obtained by the direct measurement technique of Ricou and Spalding (1961). Subsequently, we studied two cases of the double concentric jet: (a) with particles in the primary stream (1982b) and (b) with particles in the secondary stream (1982c). Case (b) was studied to calculate the effect of particles on outer shear layers. It is, therefore, believed that a systematic work on two-phase jets is necessary incorporating the effect of free stream velocity to closely simulate the industrial situation, Figure 1.

FEATURES OF A JET IN A COFLOWING STREAM

For a jet issuing from a nozzle, a shear layer is established between an inner region of constant velocity and reducing radius r_1 termed as the potential core and the expanding outer edge of the jet of radius r_2 where the velocity is U_H. The axial region up to the end of the potential core is called the zone of flow establishment. Following this is the transition and the fully developed region.

The dimensions of the shear layer of a two-phase jet in the initial region can be expressed as:

$$\frac{r_1}{r_o} = 1 - \frac{Kx}{r_o} \tag{1}$$

$$\frac{r_2}{r_o} = 1 + \frac{Mx}{r_o} \tag{2}$$

$$\frac{b}{r_o} = \frac{Nx}{r_o} \tag{3}$$

ANALYSIS

The particles affect the jet development. The effect can be estimated using the boundary layer theory of Abramovich (1963).

The slope of the inner edge of the boundary layer is given by Eq. 4 for a 2-D homogeneous jet and is used to approximate the actual system since it is amenable to mathematical analysis.

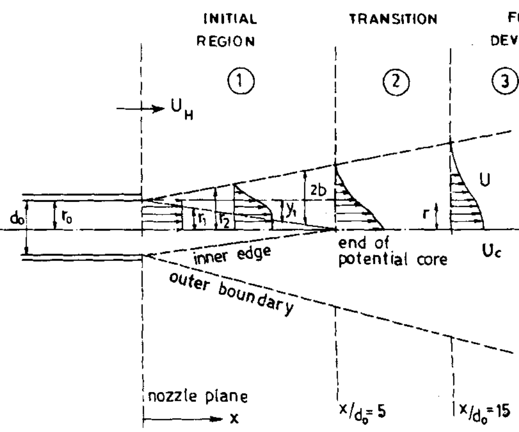

Figure 1. Development of an axisymmetric two-phase jet.

Zone 1: Initial region

Zone 2: Transition region

Zone 3: Fully developed region

Correspondence concerning this paper should be addressed to V. Subramanian at 12 Shree Ram Prasad, Bhaudaji Road Matunga, Bombay 400019, India.

R. Ganesh is presently at Virginia Polytechnic Institute and State University, Blacksburg, VA 24061.

$$\frac{y_1}{2b} = \frac{1}{2b}\int_{r_1}^{r_2} \frac{\rho}{\rho_o}\frac{U}{U_o}\left(\frac{U - U_H}{U_o - U_H}\right) dr$$

$$= \tfrac{1}{2}\int_0^2 \frac{\rho}{\rho_o}[m(1-f)f + f^2]d\eta \quad (4)$$

where

$$m = \frac{U_H}{U_o} \text{ and } \frac{\rho}{\rho_o} = \frac{1+\psi}{1+\psi_o} \quad (5)$$

Assuming that the velocity distribution follows a cosine profile in the boundary layer

$$f = \frac{U - U_H}{U_o - U_H}$$

$$= 1 - \mathrm{Sin}^2\frac{\pi\eta}{4} \quad (6)$$

$$\eta = \frac{r - r_1}{b} = \frac{r - r_1}{\tfrac{1}{2}(r_2 - r_1)} \quad (7)$$

Assuming that the concentration distribution is linear in the boundary layer

$$\eta = \frac{2(\psi_o - \psi)}{\psi_o} \quad (8)$$

Equation 4 now becomes

$$\frac{y_1}{2b} = \frac{0.38 + 0.12m + (0.29 + 0.06\,m)\psi_o}{(1+\psi_o)} \quad (9)$$

and

$$\frac{db}{dx} = \left[0.135\frac{(1+\rho_a/\rho_o)}{2}\right]\left[\frac{(1-m)}{1+(\rho_a/\rho_o)m}\right] \text{ for } m < 1 \quad (10)$$

where

$$\frac{\rho_a}{\rho_o} = \frac{1}{1+\psi_o} \quad (11)$$

Therefore

$$\frac{db}{dx} = \frac{0.0675(1-m)(2+\psi_o)}{(1+m+\psi_o)} \quad (12)$$

and

$$\frac{y_1}{r_o} = \frac{0.135(1-m)(2+\psi_o)}{(1+m+\psi_o)(1+\psi_o)}[0.38 + 0.12\,m + (0.25 + 0.06\,M)\psi_o]x/r_o \quad (13)$$

This gives

$$K = \frac{0.135(1-m)(2+\psi_o)}{(1+m+\psi_o)(1+\psi_o)}[0.38 + 0.12\,m + (0.29 + 0.06\,m)\psi_o] \quad (14)$$

$$M = \frac{0.135(1-m)(2+\psi_o)}{(1+m+\psi_o)(1+\psi_o)}[0.62 - 0.12\,m + (0.71 - 0.06\,m)\psi_o] \quad (15)$$

$$N = \frac{0.0675(1-m)(2+\psi_o)}{(1+m+\psi_o)} \quad (16)$$

Also

$$K + M = 2N$$

ENTRAINMENT

From the principle of conservation of mass, the entrainment of ambient fluid in the initial region of a jet can be written as

$$E = \frac{\int_0^{r_1} 2\pi r U dr + \int_{r_1}^{r_2} 2\pi r(\Delta U + U_H)dr}{\pi r_o^2 U_o} - 1 \quad (18)$$

$$= (1-m)\left(\frac{r_1}{r_o}\right)^2 + m\left(\frac{r_2}{r_o}\right)^2 + [2(1-m)/r_o^2][b^2F_1 + bF_2 r_1] - 1 \quad (19)$$

where,

$$\Delta U = U - U_H \quad (20)$$

F_1 and F_2 are taken to be 0.59 and 1 for a cosine velocity distribution.

Substituting Eqs. 1, 2, 3, 14, 15, 16 and 20 into Eq. 17,

$$E = A\left[\frac{x}{r_o}\right] + B\left[\frac{x}{r_o}\right]^2 \quad (21)$$

where

$$A = -2K(1-m) + 2mM + 2(1-m)N \quad (22)$$

$$B = (1-m)K^2 + mM^2 + 1.18N^2(1-m) - 2NK(1-m) \quad (23)$$

EXPERIMENTAL DETAILS

The sand/air jet issued vertically downwards from a 25.4 mm tube having a length/diameter ratio equal to 50 into a perspex chamber 150 mm in diameter and 300 mm long at a velocity of about 15 m/s as shown in Figure 2.

The moving stream was introduced through a 25.4 mm inlet as shown in Figure 2 and a honeycomb section was used to straighten the velocity profile.

The chamber had accesses at regular axial distances with rubber stoppers for accommodating the probe. A dial-type vernier attachment permitted positioning of the probe at any radial distance with an accuracy of 0.025 mm. Measurements of the velocity were made using a total head tube and the particle concentration was measured using an isokinetic sampling probe. Readings were taken for particle concentrations $\psi_o = 0, 1$ and $m = 0, 0.5$. Particles of sand of mean diameter 166 μm (size range 150–180 μm) and of density 3.4×10^3 kg/m^3 were used.

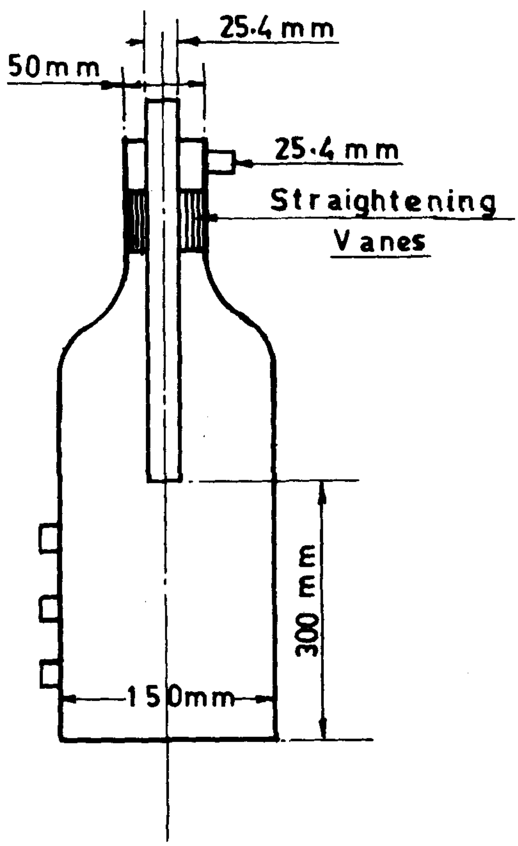

Figure 2. Experimental setup.

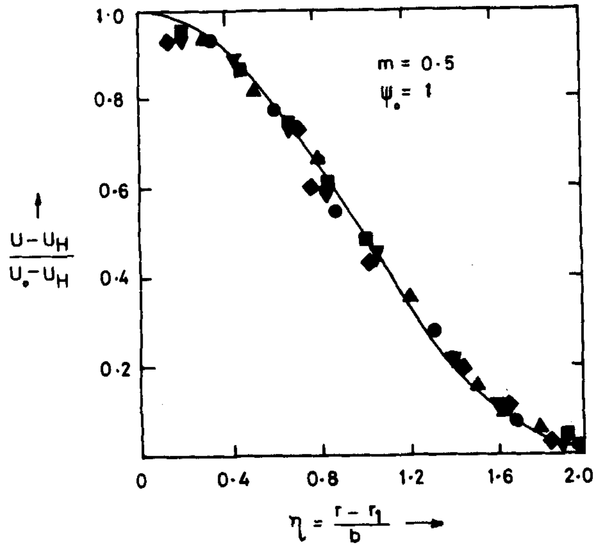

Figure 3. Similarity in velocity distribution.

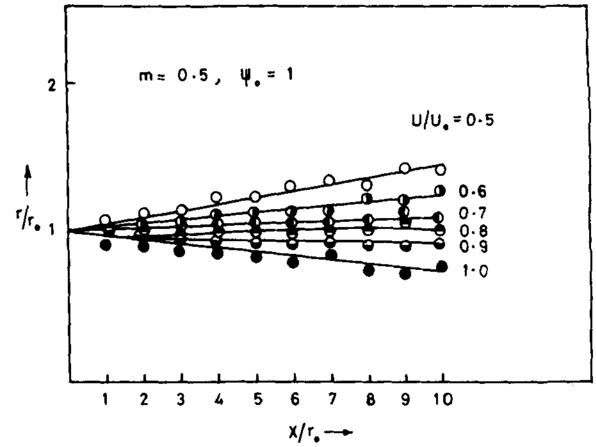

Figure 5. Equal dimensionless velocity lines for a two-phase jet in a coflowing stream.

U/U_o

0.5 ○ Experimental Data

0.6 ◑

0.7 ◐

0.8 ⊖

0.9 ◒

1.0 ●

RESULTS

The two-phase jet expansion is affected by the movement of the free stream and this influences the entrainment. Similarity of velocity, Figure 3, and particle concentration, Figure 4, distributions is experimentally observed in the annular shear layer; the cosine and straight line relationships respectively represent these profiles. Experimental points are plotted for $x/r_o = 2,4,6,10$.

The experimental values of r_1 and b have been used to reduce the data, Figures 3 and 4. Equations 1 and 3 give theoretical values of r_1 and b respectively. For $\psi_o = 1$ and $m = 0.5$, Eq. 1 predicts a slope for the potential core equal to 0.30 whereas the experimental value of K is 0.32 which gives a 6% difference between the experiments and predictions.

The slope of the half width line calculated for this case is 0.04 whereas the experimental value is 0.038.

Figure 5 shows equal dimensionless velocity lines for a two-phase jet for $m = 0.5$ and $\psi_o = 1$. The equal dimensionless velocity lines have been determined from Eqs. 3, 14, 15 and 16, and the following equations:

$$\frac{U}{U_o} = m + (1 - m)1 - \text{Sin}^2 \frac{\pi\eta}{4} \tag{24}$$

$$r = r_1 + b\eta \tag{25}$$

The experimental data are shown in Figure 5 and are seen to be fairly close to the estimates.

Figure 6 shows the entrainment (Eq. 21) in single- and two-phase jets as a function of the axial distance. Calculated values of entrainment from the experimental velocity distribution fall close to the estimates obtained based on Abramovich's (1963) theory. The curves are plots of Eq. 21 using Eqs. 22 and 23 for $m = 0.5$ and $\psi_o = 1$. The experimental points in Figure 6 have been determined from the experimental values of velocity and radii using Eq. 18 for the entrainment. It is seen that particles reduce the entrainment. This is a consequence of particle effects on turbulence. This is in agreement with our earlier work (1982a) and with the results of Levy and Lockwood (1981) for a similar system.

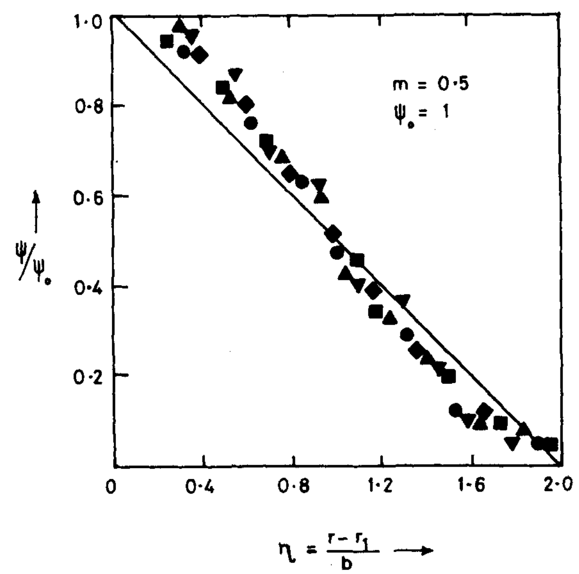

Figure 4. Similarity in concentration distribution

x/r_o

2 ● Experimental Data

4 ▲

6 ■

8 ◆

10 ▼

DISCUSSION

The mixing characteristics of the jet are affected by particles. There is a dampening of turbulence due to the presence of particles which leads to reduced mixing. The entrainment by a particle-laden jet is therefore lower.

The present work provides an expression for calculating the effect of the wind velocity on the mixing and spreading of particles. These calculations are extremely useful for pollution minimization in fields indicated earlier.

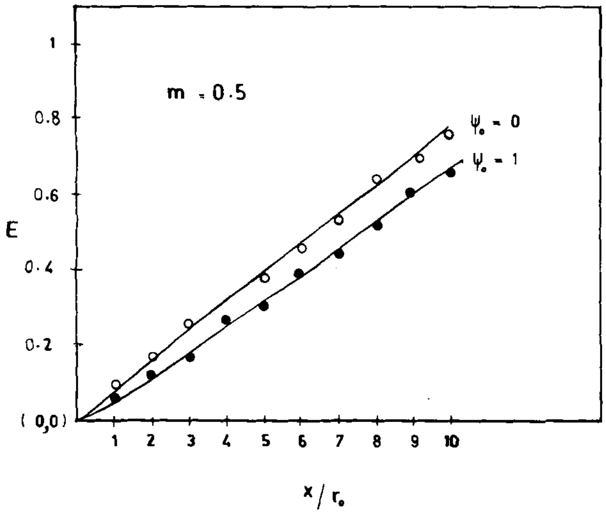

Figure 6. Entrainment in single- and two-phase jets in a coflowing stream.

$m = 0.5$

ψ_o = ○ Experimental data

$\psi_o = 1$ ●

For systems with relaxation times large compared with the Lagrangian time scale the particles will fall through the jet thus invalidating Eq. 4 and the subsequent analysis.

In the area of combustion one comes across pulverized fuel issuing into the combustion chamber and a secondary air jet issuing either from an annulus or as multiple jets. It is worthwhile to point out here that a concentric jet situation is a complex case of a jet issuing into a general free stream. The initial region of a concentric jet may be imagined to behave like a jet in a coflowing stream in the inner mixing region. Thus, our study is a first step in arriving at an expression for a more complex situation occurring in a pulverized fuel furnace.

ACKNOWLEDGMENT

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NOTATION

A,b = constants in the entrainment expression, Eq. 21
$b,2b$ = length scale (m), boundary layer thickness (m)
E = entrainment
F_1,F_2 = velocity integrals, defined by Eq. 19
f = velocity function, defined by Eq. 6
K,M,N = constants; slopes of the inner and outer edge of the shear layer, and the half width line, respectively, defined by Eqs. 14, 15 and 16
m = velocity ratio, defined by Eq. 5
r_1 = radius of potential core (m)
r_2 = radius of jet edge (m)
r_o = radius of jet exit (m)
d_o = diameter of jet exit (m)
U_H = velocity of free stream (m/s)
U_o = velocity at jet exit (m/s)
U = velocity (m/s)
U_c = center line velocity (m/s)
x = axial distance from jet exit (m)
y_1 = radial distance, Figure 1

Greek Letters

ρ = density (kg/m^3)
ρ_o = density at jet exit (kg/m^3)
ρ_a = ambient air density (kg/m^3)
ψ = concentration of particles (kg particles/kg air)
φ_o = initial particle concentration (kg particles/kg air)
η = dimensionless distance

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